LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

# PUZZLE OF THE WEEK (3/16/2017 - 3/22/2017)

**Problem:** Find, with proof, the value of the following sum.

$$\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \frac{1}{4\cdot 5} + \frac{1}{5\cdot 6} - ...$$

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**Solution:** The value of the sum is $-1 + \ln(4)$.

Using the partial fraction decomposition

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

we see that

$$\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - ... = -1 + 2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}.$$

The latter series arises from the Taylor expansion $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}x^n$ near $x = 0$. The radius of convergence of this series is equal to 1 but by Abel's Theorem the stated equality holds at the end points of the interval of convergence so long as the series converges there; hence

$$\ln(2) = \ln(1+1) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}.$$

This completes the justification of our result.