LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

PUZZLE OF THE WEEK (3/2/2017 - 3/8/2017)

Problem: Let D, E and F denote the midpoints on the sides BC, CA and AB of the triangle ΔABC. Find, with proof, the value of

$$\overrightarrow{CF} \cdot \overrightarrow{AB} + \overrightarrow{AD} \cdot \overrightarrow{BC} + \overrightarrow{BE} \cdot \overrightarrow{CA}.$$

Solution: The value in question is $\vec{0}$.

This follows from the fact that $\overrightarrow{CF} = \frac{1}{2}(\overrightarrow{CA} + \overrightarrow{CB})$, $\overrightarrow{AD} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$ and $B\vec{E} = \frac{1}{2}(\overrightarrow{BA} + \overrightarrow{BC})$. Due to linearity of the dot-product the expression at hand is equal to

$$\frac{1}{2}\left(\overrightarrow{CA} \cdot \overrightarrow{AB} + \overrightarrow{CB} \cdot \overrightarrow{AB} + \overrightarrow{AB} \cdot \overrightarrow{BC} + \overrightarrow{AC} \cdot \overrightarrow{BC} + \overrightarrow{BA} \cdot \overrightarrow{CA} + \overrightarrow{BC} \cdot \overrightarrow{CA}\right)$$
$$= \frac{1}{2}\left(\overrightarrow{CA} \cdot \overrightarrow{AB} - \overrightarrow{BC} \cdot \overrightarrow{AB} + \overrightarrow{AB} \cdot \overrightarrow{BC} - \overrightarrow{CA} \cdot \overrightarrow{BC} - \overrightarrow{AB} \cdot \overrightarrow{CA} + \overrightarrow{BC} \cdot \overrightarrow{CA}\right) = \vec{0}$$