Problem: Suppose \(a_1, a_2, \ldots, a_{2017}\) and \(b_1, b_2, \ldots, b_{2017}\) are two permutations of the set of numbers \(1, 2, 3, \ldots, 2017\). Find, with proof, the minimum value of
\[
a_1 b_1 + a_2 b_2 + \ldots + a_{2017} b_{2017}.
\]

Solution: The minimum value is
\[
1 \cdot 2017 + 2 \cdot 2016 + 3 \cdot 2015 + \ldots + 2016 \cdot 2 + 2017 \cdot 1 = 1369657969.
\]
Without loss of generality we may assume that (in that particular order) we have
\[
(a_1, a_2, \ldots, a_{2017}) = (1, 2, \ldots, 2017).
\]
The claim is that the minimum value of the expression \(a_1 b_1 + \ldots + a_{2017} b_{2017}\) is then obtained for
\[
(b_1, b_2, \ldots, b_{2017}) = (2017, 2016, \ldots, 1).
\]
Suppose the opposite. Then there exists a pair on integers \(1 \leq i < j \leq 2017\) such \(b_i < b_j\). Let \(b'_1, b'_2, \ldots, b'_{2017}\) be the permutation obtained from \(b_1, b_2, \ldots, b_{2017}\) by swapping \(b_i\) and \(b_j\). The difference between \(a_1 b_1 + \ldots + a_{2017} b_{2017}\) and \(a_1 b'_1 + \ldots + a_{2017} b'_{2017}\) is then equal to
\[
a_i b_i + a_j b_j - a_i b'_i - a_j b'_j = ib_i + jb_j - ib_j - jb_i = (i - j)(b_i - b_j) > 0.
\]
In other words, the value of \(a_1 b'_1 + \ldots + a_{2017} b'_{2017}\) would be smaller than \(a_1 b_1 + \ldots + a_{2017} b_{2017}\). Contradiction.