LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

PUZZLE OF THE WEEK (1/26/2017 - 2/1/2017)

Problem: In a convex polyhedron with m triangular faces (and possibly faces of other shapes), exactly four edges meet at each vertex. Find the minimum possible value of m, and justify your claim.

Solution: The smallest attainable value is $m = 8$.

Let V, E and F denote the numbers of vertices, edges and faces of the polyhedron; by the Euler Formula we have $V - E + F = 2$. Since each vertex corresponds to fours edges and since each edge connects two vertices, we must have $4V = 2E$, i.e $V = E/2$. Likewise, since there are $F - m$ faces bounded by at least 4 if not more edges, counting edges bounding each given face ultimately yields:

$$3m + 4(F - m) \leq 2E, \quad \text{i.e} \;\; F \leq E/2 + m/4.$$

Inserting into Euler's Formula we get

$$2 \leq m/4 \;\; \text{i.e.} \;\; m \geq 8.$$

That $m = 8$ is achievable is evident from the example of the regular octahedron.