LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

SOLUTION OF THE PUZZLE OF THE WEEK

(11/23/2016 - 11/29/2016)

Problem: How many different pairs of integers (x, y) are there with $x^2 - y^2 = 2016$? Justify your claim.

Solution: There are 48 such pairs.

Note that $x^2 - y^2 = 2016$ can be factored as

$$(x - y)(x + y) = 2^5 \cdot 3^2 \cdot 7.$$

Since integers $x + y$ and $x - y$ are of the same parity, solutions (x, y) have to be such that $x + y$ and $x - y$ are both even. By considering prime factorizations of $x + y$ and $x - y$, we see that solutions of our equation are in $1 - 1$ correspondence with integer divisors of

$$2^3 \cdot 3^2 \cdot 7.$$

There are $(1 + 3)(1 + 2)(1 + 1) = 24$ positive divisors of $2^3 \cdot 3^2 \cdot 7$. For our problem we need to include the possibility of negative divisors as well; thus our problem has 48 solutions.