LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

# SOLUTION OF THE PUZZLE OF THE WEEK

(11/9/2016 - 11/15/2016)

**Problem:** One day before noon it starts snowing and continues at a constant rate throughout the day. At noon a snowplow begins to clear a road and, moving a constant volume of snow per unit of time, clears two miles during the first hour of operation and one mile during the second hour. When did it start snowing?

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**Solution:** $11:22:55$.

We let $t = 0$ denote noon. Let $h(t)$ denote the height of snow at time $t$; by assumption we have

$$\frac{dh}{dt} = C_1.$$

It follows that $h(t) = C_1(t - t_0)$ where $t_0 < 0$ is the moment when it started snowing. Let $v(t)$ denote the linear speed of the snowplow. The rate of removal of snow (that is: the volume of removed snow per unit of time) is equal to

$$w \cdot h(t) \cdot v(t)$$

where $w$ denotes the width of the road. By assumption the latter is constant; therefore, we have that $h(t)$ and $v(t)$ are inversely proportional:

$$v(t) = \frac{C}{h(t)} = \frac{C}{C_1(t - t_0)} = \frac{C'}{t - t_0}.$$

Finally, assuming time is measured in hours and length is measured in miles, we have

$$\int_0^1 v(t)\,dt = 2\int_1^2 v(t)\,dt = 2.$$

Upon canceling $C'$ we obtain

$$\int_0^1 \frac{dt}{t-t_0} = 2\int_1^2 \frac{dt}{t-t_0}.$$

From here we get the equation

$$\ln\left(\frac{1-t_0}{0-t_0}\right) = 2\ln\left(\frac{2-t_0}{1-t_0}\right),$$

or equivalently

$$(t_0-1)^3 = t_0(t_0-2)^2.$$

After simplification the equation becomes

$$t_0^2 - t_0 - 1 = 0.$$

The only negative solution of this equation is $t_0 = \frac{1-\sqrt{5}}{2}$ hours. In terms of minutes this time is equivalent to (the negative of ) 37 minutes and 5 seconds. Thus, it started snowing at $11:22:55$.