LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

SOLUTION OF THE PUZZLE OF THE WEEK

(9/14/2016 - 9/20/2016)

Problem: Find the largest integer n such that 2^n divides the product

$$2017 \cdot 2018 \cdot 2019 \cdot ... \cdot 4031 \cdot 4032.$$

Justify your claim.

Solution: The largest such is $n = 2016$.

This problem is the special case of the following general problem:

$$(n+1)(n+2)....(n+n)$$

is divisible by 2^n but not divisible by 2^{n+1}. Amongst integers $n+1$, $n+2$, ..., $2n$ there are $n - \lfloor \frac{n}{2} \rfloor$ integers divisible by 2, $\lfloor \frac{n}{2} \rfloor - \lfloor \frac{n}{4} \rfloor$ integers divisible by 4,... $\lfloor \frac{n}{2^{i-1}} \rfloor - \lfloor \frac{n}{2^i} \rfloor$ integers divisible by 2^i, etc. Thus the highest exponent k for which $(n+1)(n+2)....(n+n)$ is divisible by 2^k is:

$$k = \left(n - \lfloor \tfrac{n}{2} \rfloor\right) + \left(\tfrac{n}{2} - \lfloor \tfrac{n}{4} \rfloor\right) + ... + \left(\lfloor \tfrac{n}{2^{i-1}} \rfloor - \lfloor \tfrac{n}{2^i} \rfloor\right) + ... = n.$$