LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

SOLUTION OF THE PUZZLE OF THE WEEK

(9/7/2016 - 9/13/2016)

Problem: Consider the sequence of integers for which the n-th term of the sequence is equal to the perfect square closest to n:

$$1, 1, 4, 4, 4, \ldots$$

Which, if any, values appear in this sequence exactly 2016 times? Justify your claim.

Solution: The value of 1008^2 appears in the sequence exactly 2016 times.

To see this let a_n (for $n \geq 1$) denote the n-th term of our sequence. Observe that the closest perfect square to a number of the form $n^2 - n$ is n^2:

$$(n^2 - n) - (n-1)^2 = n + 1 > n = n^2 - (n^2 - n).$$

Likewise, the closest perfect square to a number of the form $n^2 + n$ is also n^2. Thus, for all $-n \leq i \leq n$ we have

$$a_{n^2+i} = n^2.$$

It follows that each perfect square n^2 appears in the sequence $2n$ times. Specifically, 1008^2 appears in the sequence 2016 times.