LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

PUZZLE OF THE WEEK (3/16/2016 - 3/29/2016)

Problem: Find, with proof, all positive integers n for which $1!+2!+3!+...+n!$ is a perfect square.

Solution: The only solutions are $n = 1$ and $n = 3$.

To see this, observe that

$$1! = 1, \quad 1! + 2! = 3, \quad 1! + 2! + 3! = 9, \quad 1! + 2! + 3! + 4! = 33.$$

Therefore, $n = 1$ and $n = 3$ are the only solutions with $n \leq 4$. For $n \geq 5$ we have that $n!$ is divisible by 10, meaning that the last digit of $1! + 2! + ... + n!$ must be 3. On the other hand, the last digit of a perfect square must be 0, 1, 4, 5, 6 or 9. Thus, $1! + 2! + ... + n!$ cannot be a perfect square for $n \geq 5$.