LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

PUZZLE OF THE WEEK (2/24/2016 - 3/1/2016)

Problem: Let us agree to call a positive integer *subprime* whenever the number of its decimal digits equals to the number of its distinct prime factors. (So, for example, 12 is subprime while 25 is not.) Are there finitely or infinitely many subprime numbers? Justify your claim.

Solution: There are finitely many subprime numbers. Specifically, if m denotes the number of primes which are less than 100 then there can be no more than 10^{2m} subprime numbers.

To see this suppose that n is a subprime number with d decimal digits. It suffices to show that $d \leq 2m$. Assume the opposite: that $d > 2m$. Amongst the first d primes there are $d - m$ primes which have 2 or more digits. Thus, the product of the first d primes is more than

$$10^{2(d-m)} > 10^d \cdot 10^{d-2m} > 10^d.$$

Since n (being subprime) is at least equal to the product of the first d primes it would have to have at least as many decimal digits as 10^d. In other words, n would have at least $d + 1$ digits. Contradiction.