LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

PUZZLE OF THE WEEK (2/3/2016 - 2/9/2016)

Problem: A country has N major cities with an airport. For each city there is a direct flight to exactly 3 other cities. Furthermore, one is able to fly from every city to every other city with at most one layover. What is the maximum value of N? Justify your claim.

Solution: If we assume that each direct flight has a return flight, then the maximum value of N is 10. If it is assumed that out-bound flights are not reciprocated with a return flight, then the maximum value of N is 12. The latter is a considerably more difficult problem which requires some knowledge of graph theory. (Interested students should look up *almost Moore digraphs.*) The solution presented below assumes that each direct flight has a return flight.

A city A is connected to 3 other cities: B, C and D. Each of the cities B, C and D is connected to two cities other than A:

$$B_1, B_2, C_1, C_2, D_1, D_2.$$

It is not clear that all the ten cities mentioned thus far are distinct. However, since citizens of A should be able to reach every other city with at most one layover we know that there cannot be any cities other than

$$\{A, B, C, D, B_1, B_2, C_1, C_2, D_1, D_2\}.$$

In particular, we conclude that $N \leq 10$.

The map on the following page shows that $N = 10$ can indeed be achieved: