LEWIS AND CLARK COLLEGE
Department of Mathematical Sciences

SOLUTION OF THE PUZZLE OF THE WEEK
(9/21/2016 - 9/27/2016)

Problem: Let $f(x)$ be some continuously differentiable function of real variable with $f(0) = f(1) = 0$. Does there have to exist some real number c between 0 and 1 such that $f'(c) + f(c) = 0$? Justify your claim.

Solution: Yes, such a c has to exist.
Consider the auxiliary function $g(x) = e^x f(x)$. We now have $g(0) = g(1) = 0$. By the Mean Value Theorem there is some c between 0 and 1 such that

$$g'(c) = 0.$$

By the Product Rule we have $g'(x) = e^x(f(x) + f'(x))$. Since $e^c \neq 0$ the condition $g'(c) = 0$ simply becomes $f(c) + f'(c) = 0$.